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Influence of the layup configurations of the laminate on the buckling and post-buckling states of the plate element

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ABSTRACT: Symmetrical laminates do not exhibit a tendency to twist as a result of the cooling phase during fabrication. For these reasons, symmetrical composites are widely used by designers in engineering practice. These materials are better "tailored" to the demands of optimal designs. In general, designers evade non-symmetric sequences of layers in this type of structures. Given a wide range of application possibilities for non-symmetric and general laminates, it is worth focusing on their advantages. Such laminates are very interesting in terms of passive structures, where in-plan load induces out-of-plane deformation. In order to solve this problem, it is necessary to describe the behaviour of thin-walled structures made of general laminates under any loads. The main disadvantage of general laminates is connected with mechanical couplings. There are many types of couplings which depend on the stacking sequence of laminate layers. The presence of mechanical couplings causes that their mechanical properties will be changed both in buckling and post-buckling states. In the literature one can find few studies on the effect of non-symmetrical couplings on the behaviour of composite structures under compression.

KEYWORDS: buckling, laminate, mechanical coupling, CLT

1. Introduction

In general, designers evade non-symmetric sequences of layers in this type of structures. This does not mean, however, that it is impossible to produce structures made of non-symmetric laminates, the so-called HTCS laminates (i.e. hygro-thermally curvature-stable laminates). Winckler [1] was the first to present the idea of HTCS laminates. Such laminates are developed to be immune to hygro-thermal warping during production. Li et al. discussed the problem of optimal plates made of both HTCS laminates with extension-twist coupling [2] and HTCS laminates with extension-shear coupling [3]. Haynes and Armanios [4] analysed the HTCS laminates that produce maximum extension-twist coupling. They showed a few stacking sequences of laminate layers to demonstrate improvement in coupling. York [5] discussed a number of stacking sequence configurations of layers for HTCS laminates and general laminates with different types of mechanical coupling. A general theory of the HTCS laminates is presented in Vannucci et al.[6].

2. Problem formulation

Let us consider the problem of static buckling of a thin-walled plate element made of general laminate subjected to in-plane uniaxial load. A geometry of the plate is shown in Fig. 1.

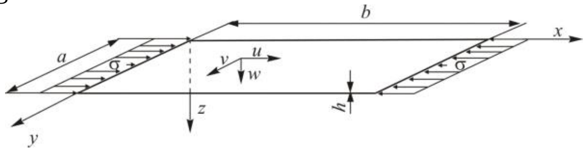

Fig. 1. Geometry of the analyzed plate

It is assumed that the Kirchhoff-Love theory is valid [7]. The general laminate will be composed of thin layers with different thickness and/or orientation of fibres. The fibre orientation will be different in each layer, providing the structure with the desired properties in required directions. Both individual plies and the whole composite will be treated as a continuum. The classical lamination theory (CLT) is used [8]. The laminate's constitutive equations have the following form:

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \{\varepsilon^o\} \\ \{\kappa\} \end{Bmatrix} \qquad (1)$$

The above equations are general and enable investigating behaviour of any laminate. These relations are simplified in particular cases of ply stacking sequences. When a composite is symmetrical, the stiffness matrices of individual plies and their thicknesses are symmetrical in relation to the mid-plane, such that the submatrix [B]=0. In a particular case of fibre orientation, not only [B]=0 but also the following terms are equal to zero: $A_{16}=A_{26}=0$ and $D_{16}=D_{26}=0$. This means that there are no mechanical couplings in the laminate, and the load-deformation interactions are simple.

In a general case of non-symmetrical composites, the mechanical couplings are possible when:

1. $A_{16}\neq0$ and $A_{26}\neq0$ (normal load – shear strain),
2. $D_{16}\neq0$ and $D_{26}\neq0$ (bending moment – twisting strain),
3. $B_{11}\neq0$, $B_{12}\neq0$ and $B_{22}\neq0$ (normal load – flexural strain),
4. $B_{66}\neq0$ (shear load – torsional strain),
5. $B_{16}\neq0$ and $B_{26}\neq0$ (normal load – twisting strain).

3. Analytical-numerical method of analysis

The Koiter's theory is founded on the asymptotic expansion of a post-buckling equilibrium path and allows for mutual interaction of nearly simultaneous buckling modes. The displacement field and sectional force field are expanded into the power series with respect to the buckling linear eigenvector amplitude (normalized with the equality condition between the maximum deflection and the thickness of the first plate– denoted as h).

The zero approximation describes the pre-buckling state, whereas the first order approximation enables determination of critical loads and the buckling modes. The second order approximation is reduced to a linear system of differential heterogeneous equations, where the right-hand sides depend on the force field and the first order displacements only. The static system of ordinary differential equations of equilibrium can be solved by the modified numerical transition matrix method, wherein the state vector of the final edge is derived from the state vector of the initial edge by numerical integration of differential equations along the circumferential direction formulae by means of the Godunov orthogonalization method. This method enables determining the post-buckling coefficients which can be used to describe the post-buckling equilibrium path for static load. What is important about this method is that one can use it to solve static and dynamic buckling problems simultaneously. One-mode approach can be written as:

$$\left(1-\frac{\sigma}{\sigma_1}\right)\zeta_1 + b_{111}\zeta_1^2 + c_{1111}\zeta_1^3 = \frac{\sigma}{\sigma_1}\zeta_1^* \qquad (2)$$

where: $\zeta_1 = w/h$ is the dimensionless amplitude of the buckling mode, σ_1, $\zeta_1^* = w_o/h$ are the critical stress and dimensionless amplitude of the initial deflection. Having found the solutions for the first and second-order boundary problem, the coefficients b_{111}, c_{1111} describing the post-buckling equilibrium path are determined.

4. Selected numerical results

Detailed calculations were performed for a square plate, simply-supported along all edges made of a general laminate under a uniform uniaxial compression load (Fig. 1). In the buckling state for the basic mode (m=1,n=1), the lowest value of the buckling stress is observed for the laminate with extension-shearing, bending-extension, bending-shearing, twisting-extension, twisting-shearing and bending-twisting couplings. The indexes (m,n) are numbers of the half-waves parallel to the respective axes x, y of the local Cartesian system. The layup configurations of the laminate was $[(15/0)_6]_T$. The lowest buckling stress is observed for the fully isotropic laminate without coupling (denoted as FIL), while the highest value of buckling stress is observed for the laminate with bending-twisting coupling. The difference between the highest and the lowest values of the buckling stress is almost 40%. Comparably high values of buckling stress are obtained in other cases. A similar behaviour could be observed for modes (m=1,n=2) and (m=2,m=2). With higher modes (m,n), the lowest value of buckling stress is observed for the fully isotropic laminate (denoted as FIL). Low buckling stress is observed for the laminate with the following layup configuration: $[0/90/15/15_2/15/0/15/\text{-}15/90/0/90]_T$ (i.e. laminate with bending-extension, bending-shearing, twisting-extension and twisting- shearing couplings), while the highest value of buckling stress is exhibited by the laminate with the layup configuration: $[(15/\text{-}15/0)_4]_T$ (i.e. laminate with bending-extension, bending-shearing, twisting-extension, twisting-shearing and bending-twisting). The difference between the highest and the lowest values of buckling stress amounts to almost 70%. The high value of buckling stress is obtained for the laminate with the following layup configuration: $[(15/\text{-}15_2/15)_3]_T$ (i.e. laminate with bending-twisting coupling.

The post-buckling paths have the highest rigidity if the layup configuration is $[(0/90)_6]_T$ (i.e. fully isotropic laminate), $[(15/\text{-}15_2/15)_3]_T$ (i.e. laminate with bending-twisting coupling). As for other the layup configurations: $[0/90/15/15_2/15/0/15/\text{-}15/90/0/90]_T$ (i.e. laminate with bending-extension, bending-shearing, twisting-extension and twisting- shearing couplings) and $[(15/\text{-}15/0)_4]_T$ (i.e. laminate with bending-extension, bending-shearing, twisting-extension, twisting-shearing and bending-twisting couplings), the rigidity of the post-buckling paths is lower by 25%. Summing up the results, the most interesting observations can be made about the laminate with bending-twisting coupling (i.e. $D_{16}\neq 0$ and $D_{26}\neq 0$) when the system is $[(15/\text{-}15_2/15)_3]_T$ (i.e. laminate with bending-twisting coupling) because the plate element has the highest value of buckling stress, and the post-buckling rigidity does not decrease. The bending-extension, bending-shearing, twisting-extension and twisting-shearing couplings (i.e. $[B]\neq 0$) are significant, because the buckling stress increases slightly while the post-buckling rigidity significantly decreases. The extension-shearing coupling (i.e. $A_{16}\neq 0$ and $A_{26}\neq 0$) can be neglected.

5. Final remarks

The bending-twisting coupling has a significant influence on the static buckling and post-buckling states of thin-walled plate elements under compression. The plate element has the highest value of buckling stress, and the post-buckling rigidity does not decrease. The fully isotropic laminate shows low values of buckling stress, but the post-buckling paths exhibits high rigidity. The bending-extension, bending-shearing, twisting-extension and twisting-shearing couplings (i.e. $[B]\neq 0$) are significant, whereas the extension-shearing coupling can be neglected.

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