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Influence of the coupling submatrix B on the nonlinear stability of FG cylindrical panels subjected to compression

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ABSTRACT: The nonlinear stability of cylindrical panels simply supported at all edges has been discussed. Panels are made of Functionally Graded Materials (FGMs). FG panels can be treated as multilayered composite structures of transverse inhomogeneity. Such structures have the non-zero coupling submatrix B. It results in the fact that for FG plate structures, postbuckling equilibrium paths are unsymmetrically stable, and thus the system response depends on the initial imperfection sign (sense). An effect of the coupling submatrix B and the sign of initial perfections on postbuckling equilibrium paths of actual FG panels is analysed

KEYWORDS: panels, FGM, postbuckling path

1. Introduction

Since the mid 1980s, Functionally Graded Materials (FGMs) have been a relatively new class of composite materials used in numerous engineering applications. A standard functionally gradient material is an inhomogeneous composite made up of two constituents – typically of metallic and ceramic phases. The combination of ceramics with a metal component renders specific characteristics to FG structures – a better resistance to high temperature (ceramics) and good mechanical features (metal), reducing further a possibility of fracture of the whole gradient structure. These features make high temperature environments the leading application area for FG structures.

In [1], on the basis of Koiter's nonlinear theory of conservatory systems, it has been shown that FG plate structures have unsymmetrical stable postbuckling equilibrium paths. Due to the presence of the nontrivial submatrix *B*, the coupling between extensional and bending deformations exists as it is in the case of unsymmetrical laminated plates. Attention has been drawn to an effect of the imperfection sign (sense).

An influence of the transverse inhomogeneity of FG plates subjected to compression in the postbuckling state has been discussed. The transverse inhomogeneity of FG structures entails an occurrence of non-zero coupling matrix *B*, and, thus, various postbuckling equilibrium paths of real structures with imperfection.

2. Formulation of the problem

In thin-walled FG structures such as plates or shells, the volume fractions of ceramics V_c and metal V_m are described usually by a simple power law of distribution throughout the structure thickness t :

$$V_c(z) = \left(z/t + 0.5\right)^q \qquad V_m(z) = 1 - V_c(z) \qquad (1)$$

According to the rule of mixture, the properties of the functionally graded material (E – Young's modulus, ν – Poisson's ratio) can be expressed.

The commercial ANSYS software is applied in the numerical calculations. In the finite element method solution, FG plates are modelled as multilayered composite structures [2], whose graded material properties in the range of 10-40 isotropic layers are defined. After the convergence analysis, the model with twenty layers is accepted. For meshing, a shell element (SHELL181 from ANSYS library) is employed with four nodes and six degrees of freedom at each node; the total number of degrees of freedom is equal to 4056.

The initial imperfections are introduced by updating the finite element mesh with the first mode shape of the eigen-buckling solution, with an assumed magnitude corresponding to the panel thickness. The eigen-buckling analysis, where the critical load is determined despite the eigen-mode and the modal analysis, preceded the nonlinear buckling analysis.

3. Analysis of the results

Cylindrical shallow panels (Fig. 1), subjected to axial compression along the direction z and simply supported at all edges, are considered. All panels are subject to Hooke's law. A detailed analysis of the calculations is conducted for thin-walled panels with the following dimensions (Fig. 1): a=b=75.0 mm, t=1.0 mm, R=500.0 mm.

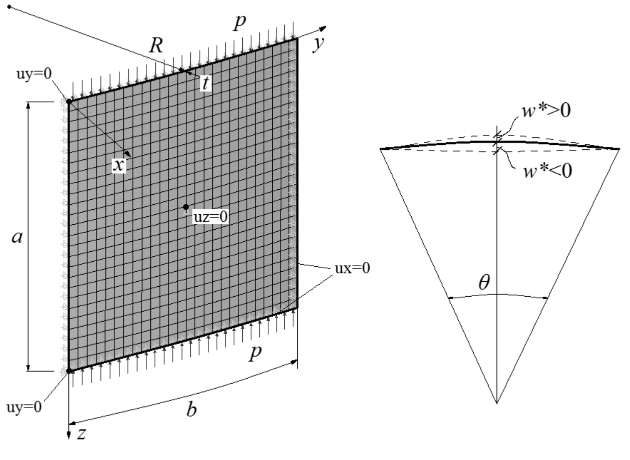

Fig. 1. Cylindrical shallow panels

Two various materials of the panels are made, namely:

- isotropic material (Aluminium) with the following material properties: Young's modulus E=79.28 GPa and Poisson's ratio ν =0.3268;

- Al-TiC functionally graded material for the fraction exponent (1) $q=1.0$ [1]. The component material properties of Al-TiC functionally graded materials are as follows: Al Young's moduli: E_m=69 GPa, TiC Young's moduli: E_c=480 GPa, Al Poisson's ratio: ν_m=0.33, TiC Poisson's ratio: ν_c=0.20.

Three variants of the panels are considered, namely:

- variant I – isotropic panel (the so-called reference variant), IP;
- variant II – FG panel (ceramics lower surface of the cross-section, metal – external one), PCM;
- variant III – FG panel (metal lower surface of the cross-section, ceramics – external one), PMC.

The isotropic panel is homogenously transversely symmetrical, whereas the FG panels are transversely inhomogeneous. Two variants of FG columns are assumed. Two cases of manufacturing of such columns are possible. The ceramic surface is resistant to high temperatures. Variant II protects the internal surface of the panel against high temperature, whereas variant III protects the external surface.

The assumed boundary conditions and a division into finite elements are presented in Fig. 1.

Table 1 shows critical values for three variants of the panels under consideration. Postbuckling equilibrium paths for ideal and real panels are presented in Figs. 2-4. These figures depict an influence of the external loading on the panel maximal deflection *w*. Various values of initial imperfections of the FG panels are studied: w^*=-0.02t; 0.02t; -0.2t and 0.2t (where t is the panel thickness). In the case of the isotropic panel (variant I), only two first values of imperfection are assumed. The considered signs (senses) of imperfection are shown in Fig. 1 accordingly to the used system of coordinates. For the isotropic panel IP (Fig. 2), one can see that the imperfection w^* sign exerts a considerable influence on different postbuckling equilibrium paths, i.e., on various signs (senses) of the deflection *w* values. For the PCM panels, the imperfection w^* sign (sense) does not affect the deflection *w* direction towards the panel centre (Fig. 3). Also for the PMC panels (Fig. 4), the imperfection w^* sign (sense) does not influence the deflection *w* direction outside the panels. In Fig. 4, the curves for the imperfection w^*=-0.2t; 0.2t overlap, identically as for w^*=-0.02t; 0.02t. As one can easily observe in Figs. 3 and 4, the imperfection sign (sense) does not affect the displacements *w* for the PCM and PMC panels, however the absolute quantity w^* exerts such an influence.

Table 1. Critical load N_{cr} for three variants of the panel

Critical load	Variant I - IP	Variant II - PCM	Variant III - PMC
N_{cr} [kN]	4.794	13.478	14.110

4. Conclusions

It can be seen that the sign of imperfection exerts an influence on the postbuckling equilibrium path of the FG panels. This effect can be easily explained by the fact that the coupled submatrix *B* is different from zero for the FG panels. Similar effects can be observed also for composite panels when the submatrix *B* is non-zero.

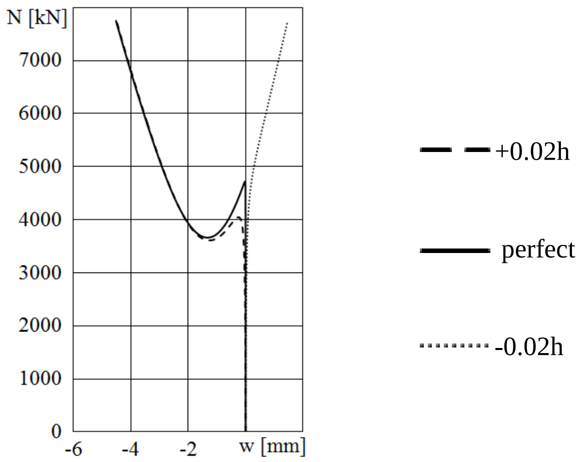

Fig. 2. Isotropic panel IP

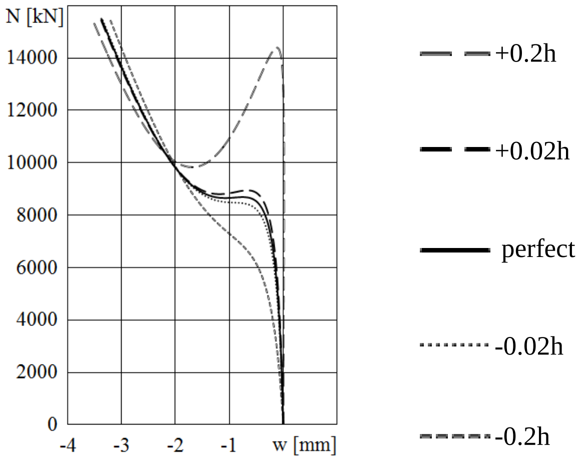

Fig. 3. FG panel PCM

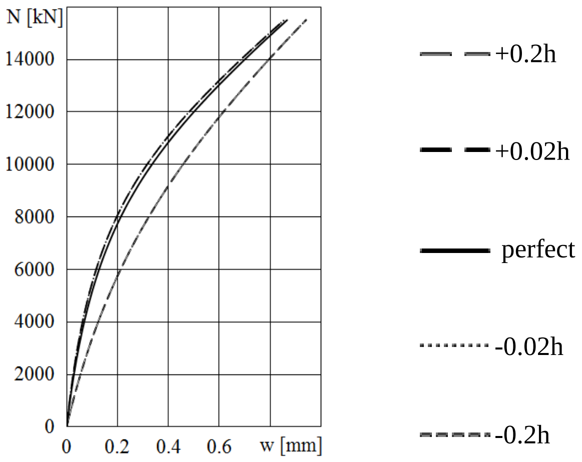

Fig. 4. FG panel PMC

References

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