Three-dimensional numerical modelling of head conduction in curved laminated glass

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ABSTRACT: The two-dimensional (2D) numerical model is applied for full three-dimensional (3D) heat transport in curved laminated glass. The laminated glass consists of two or three glass panes boned by thin layers of polymer. In the numerical model each layer (glass or polymer) is separately approximated in the thickness direction, in consequence the volume integrals can be expressed by integrals along the mid-surface of the laminated glass. Subsequently, the 3D mid-surface can be transformed to 2D domain. In the 2D numerical model the full 3D physical model of the curved structure is applied. In the postprocessing, tailored to the method, full 3D results can be graphically observed. The method is illustrated with examples where the correctness and effectiveness of the presented method are proved.

KEYWORDS: curved laminated glass, reference domain, head conduction

1. Introduction

This work deals with three-dimensional (3D) numerical modelling of heat conduction in curved laminated glass (LG). The numerical model is based on two-dimensional (2D) finite element method (FEM), but the numerical model 'keeps' full 3D physical model. The 2D numerical model is flexible and is relatively easy to use and it can be used in laminated structure with arbitrary number of layers and the layers can be thick as well as very thin. The number of layers and the their properties (i.e. thickness, thermal material properties) are just the input to the 2D model. In spite of fact that 2D mesh is used in the analysis the full three-dimensional results are obtained.

The laminated glass is a complicated structure for numerical modelling due to the fact that it consists of two or more glass paned boned by thin layers of polymer: polyvinyl butyral (PVB) or Ethylene Vinyl Acetate (EVA) There is also a great difference in thermal conductivity between glass and the polymers which are good insulators in comparison to glass [2, 1].

In spite of the fact that the considered laminated glass is not planar the numerical model is based on planar finite mesh. It is obtained firstly by reduction of the numerical model to the 3D curved surface. Subsequently, the surface is transformed to planar 2D domain, which is then discretized by 2D FE mesh. It should be stressed here that in contrary to standard approach (i.e. plates and shells models) the physical model is three-dimensional all the time, only the numerical description is reduced to planar 2D analysis.

LG are widely used in most civil or automotive engineering structures and their thermal or mechanical analysis is supported by a number of mathematical theories and plenty of software implementations. The numerical modelling are all the time in the scope of interest of many papers, e.g. [5, 1].

The 3D analysis of laminated composite plate is rather rarely performed due to problems connected with discretisations. In that case the spatial finite elements have to be adjusted to the laminates and it may lead to ill- conditioning elements with high aspect ratio. The 3D plate analysis by FEM sometimes is considered but when three-dimensional effects need to be modelled [4, 3].

2. Methodology

It is assumed in this work that the heat transfer is non-stationary so time integration has to be performed and linear system of equations need to be sequentially solved to get temperature increments. Each laminate layer in transverse direction is approximated by first, second or fourth order approximations. Afterwards, the global system of equation for the whole structure comes from assembling procedure over all layers. In the presented approach very thick layers can be combined with very thin layers without any numerical instabilities. The order of approximation in transverse direction can be adjusted to the layer thickness. For thick layer second or fourth order transverse approximation can be used while for thin layer the first order is enough.

For the sake of clarity, the analysis is presented for isotropic, homogeneous curved plate with first order transverse approximation. For multi-layered structure the same approach as for homogeneous plate is applied separately for each of the layer. Afterwards the assembling procedure is used for the global discrete system of equations. Finally, the curved layered structure can be analysed by 2D finite element mesh.

In spite of the fact that only 2D FEM is applied the character of the results are full three-dimensional. A smart postprocessing procedure needs to be applied for full 3D graphical visualisation of the results. Such a postprocessing is proposed and the temperature as well as heat flux can be graphically presented.

The presented method is illustrated by a couple of numerical examples. In these examples the curved laminated glass structures under thermal load are analysed. The laminated glass consists of two or three glass panes boned by thin layers of polymer. The correctness and effectiveness of the presented method is shown in these examples.

3. Numerical model for curved plate

The 3D homogeneous V with outer boundary S is considered. The domain has shape of curved plate with h thickness. In this domain the upper-surface S^u, mid-surface S^m and lower-surface S^l are distinguished. On S^m surface a local coordinate system is constructed defined by three orthogonal unit vectors $(\mathbf{n}, \mathbf{s}, \mathbf{r})$ normal and two tangents, respectively. The Fig. 1 illustrates the cross-section of the considered domain where the local coordinates are presented.
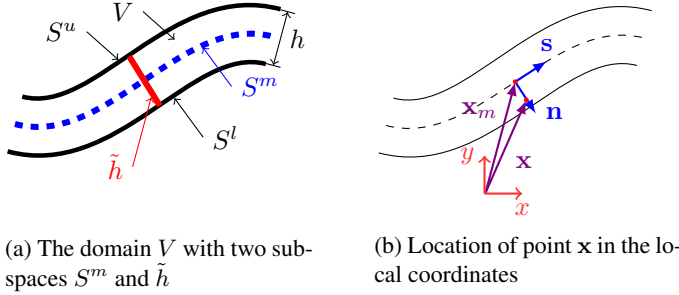

(a) The domain V with two sub-spaces S^m and $\tilde{h}$

(b) Location of point $\mathbf{x}$ in the local coordinates

Figure 1: Cross-section of domain V perpendicular to $\mathbf{r}$ with local coordinates on S^m

In the considered domain the following second order boundary value problem of heat conductivity problem is formulated

$$
\begin{aligned}
c\rho\dot{\Theta} + \operatorname{div}\mathbf{q} &= r && \text{in } V \\
\mathbf{q} &= -k\boldsymbol{\nabla}\Theta && \text{in } V, \quad \Theta = \hat{\Theta} \quad \text{on } S_\Theta \\
\mathbf{q}\cdot\mathbf{n} &= \hat{q}, && \text{on } S_q, \quad \Theta = 0 \quad \text{for } t = 0
\end{aligned}
\tag{1}
$$

where ρ is the mass density, c is the specific heat capacity, Θ is the relative temperature, $\hat{\Theta}$ is the prescribed temperature, $\hat{q}$ is the prescribed boundary heat flux, $\mathbf{q}$ the heat flux density vector, r the heat source density, k is the conductivity parameter, Additionally it is assumed that heat source and boundary values $\hat{\Theta}$ and $\hat{q}$ do not change in time.

The weak form of eq. (1) is the scalar integral equation with the scalar test function v. The volume integrals can be written as integrals over S^m and along the plate thickness. Similarly the inner product $\boldsymbol{\nabla}v\cdot\boldsymbol{\nabla}\Theta$ can be expressed by derivatives into the transverse direction and tangents to S^m surface

$$
\begin{aligned}
\boldsymbol{\nabla}v\cdot\boldsymbol{\nabla}\Delta\Theta &= \frac{\partial v}{\partial x}\frac{\partial\Delta\Theta}{\partial x} + \frac{\partial v}{\partial y}\frac{\partial\Delta\Theta}{\partial y} + \frac{\partial v}{\partial z}\frac{\partial\Delta\Theta}{\partial z} = \\
\frac{\partial v}{\partial n}\frac{\partial\Delta\Theta}{\partial n} &+ \frac{\partial v}{\partial s}\frac{\partial\Delta\Theta}{\partial s} + \frac{\partial v}{\partial r}\frac{\partial\Delta\Theta}{\partial r} = \frac{\partial v}{\partial n}\frac{\partial\Delta\Theta}{\partial n} + \bar{\boldsymbol{\nabla}}v\cdot\bar{\boldsymbol{\nabla}}\Delta\Theta
\end{aligned}
\tag{2}
$$

Regarding the backward Euler time integration the integral equation for the problem defined in eq. (1) is now as follows

$$
\begin{aligned}
&\frac{1}{\Delta t}\int_{S^m}\int_{-\frac{h}{2}}^{\frac{h}{2}} c\rho\, v\,\Delta\Theta\,\mathrm{d}n\,\mathrm{d}S + \int_{S^m}\int_{-\frac{h}{2}}^{\frac{h}{2}} k\frac{\partial v}{\partial n}\frac{\partial\Delta\Theta}{\partial n}\,\mathrm{d}n\,\mathrm{d}S \\
&+ \int_{S^m}\int_{-\frac{h}{2}}^{\frac{h}{2}} k\bar{\boldsymbol{\nabla}}v\cdot\bar{\boldsymbol{\nabla}}\Delta\Theta\,\mathrm{d}n\,\mathrm{d}S \\
&= \int_{S^m}\int_{-\frac{h}{2}}^{\frac{h}{2}} vr\,\mathrm{d}n\,\mathrm{d}S - \int_{S^q} v\,\hat{q}\,\mathrm{d}S + \int_{S^m}\int_{-\frac{h}{2}}^{\frac{h}{2}} \boldsymbol{\nabla}v\cdot\mathbf{q}^t\,\mathrm{d}n\,\mathrm{d}S
\end{aligned}
\tag{3}
$$

where $\mathbf{q}^t$ is the heat flux at previous time step.

The integrals along the plate thickness in eq. (3) can be expressed by the one-dimensional Gauss integration rules using the Gaussian points $n_1^{(2)}$, $n_2^{(2)}$ for two-point rule

$$
\int_{S^m}\int_{-\frac{h}{2}}^{\frac{h}{2}} F\,\mathrm{d}n\,\mathrm{d}S = \int_{S^m} \frac{h}{2}\left(\left(F\right)\Big|_{n=n_1^{(2)}} + \left(F\right)\Big|_{n=n_2^{(2)}}\right)\mathrm{d}S
\tag{4}
$$

where F is any integrand from eq (3).

The equation (3) can now be expressed only by integrals over the S^m surface. The reference 2D domain Γ can be now used for calculations and the results transformed to S^m surface by the following transformation

$$
T: \boldsymbol{\xi} \in \Gamma \rightarrow \mathbf{x} \in S^m
\tag{5}
$$

The integrals can now be calculated on 2D reference Γ domain instead on S^m surface. Also the derivatives can be can be easily transformed to the reference domain

$$
\bar{\boldsymbol{\nabla}}v\cdot\bar{\boldsymbol{\nabla}}\Delta\Theta = \bar{\bar{\boldsymbol{\nabla}}}v^{\mathrm{T}}\mathbf{J}_{rs}^{-1}\mathbf{J}_{rs}^{-\mathrm{T}}\bar{\bar{\boldsymbol{\nabla}}}\Delta\Theta
\tag{6}
$$

where $\bar{\bar{\boldsymbol{\nabla}}}$ is the gradient operator in Γ domain and $\mathbf{J}_{rs}$ is the appropriate Jacobian matrix.

4. Conclusions

The presented numerical model was applied to heat transport in two-pane and three-pane curved laminated glass. The thickness of the glass pane is 5–10 mm while the polymer film is only 0.38 mm thick. The non-stationary heat flow was regarded. In the analysis full 3D effects can be observed instead of the fact that a planar finite mesh is used for the analysis. The results were compared with the standard approach with 3D finite element mesh and good agreement of those two methods can be observed.

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