Simulation of instabilities in plasticity and thermo-plasticity

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ABSTRACT: The paper presents the concepts of theoretical and numerical analysis of material instabilities and induced localized deformations, in particular shear bands. Attention is focused on rate-independent plasticity, the analysis starts from small strain isothermal models and extends to large strain thermo-plasticity. The possible gradient-enhancements of the models are discussed, which result in prevention of pathological discretization sensitivity of finite element simulations. The importance of couplings is stressed and algorithmic aspects are addressed.

KEYWORDS: material instability, strain localization, finite element method, gradient-enhanced continuum, finite strain, plasticity, thermo-mechanical coupling

1. Introduction

A proper representation of localized strains is a major problem in the modelling of materials. Localization means that from a certain stage of the deformation history of a considered specimen onward, the strains grow only in narrow bands while unloading takes place in the remaining parts of the specimen [1, 2]. Typical examples of localized deformation are shear bands in soil, micro-cracking bands in a quasi-brittle material or necking in a metallic material.
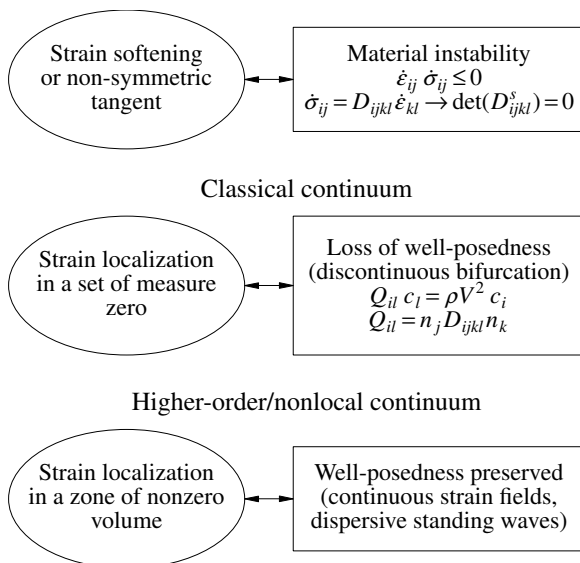

Figure 1: Consequences of material instabilities [3]

Strain localization occurs when the considered material suffers from a loss of stability at a certain level of deformation [4, 5]. When the material tangent stiffness D_{ijkl} becomes negative definite (softening is encountered) or when a non-associative plastic flow is considered (resulting in nonsymmetry of the tangent operator), ellipticity of the governing partial differential equations can be lost in statics and hyperbolicity can be lost in dynamics. This loss of well-posedness of the classical continuum model is indicated by the singularity of the acoustic tensor Q_{il}. When an eigenvalue of Q_{il} becomes negative for a softening medium, the waves cannot propagate

(wave speed V is imaginary) and a loading wave does not transform into a stationary localization wave.

The ill-posedness of the (initial) boundary value problem results in a loss of uniqueness of the solution. An infinite number of solutions can be obtained and they can involve discontinuities of the deformation gradient (strain localization in a set of measure zero), cf. Fig. 1. As a result, the numerical simulations of the phenomenon suffer from convergence problems and a pathological mesh sensitivity of the results is observed which is related to the tendency to simulate localization in the smallest volume of the material admitted by the discretization. The description must then be regularized to obtain proper results. Limiting discussion to rate-independent (inviscid) inelastic continuum, either nonlocal integral or gradient-enhanced formulation can be used.

2. Small strain isothermal plasticity

The first part of the paper contains the discussion of the above issues, assuming linear kinematic equations and isothermal condition. Attention is focused on shear band formation in geomaterials, modelled with Cam-clay gradient-dependent plasticity theory implemented in the *FEAP* package [6]. The formulation can be extended to take into account the saturation of soil pores with a fluid, leading to a two-phase model with excess pore pressure as additional fundamental unknown and the mass continuity equation. The gradient-enhancement involves the introduction of an internal length parameter and requires the solution of an additional differential equation governing the evolution of the plastic strain measure. This approach gives rise to a coupled problem and two- or three-field finite element formulation [7]. Fig. 2 compares the shear bands simulated in a vertical embankment test using classical and gradient-enhanced description. In the former case mesh-sensitive results are obtained and strains localize in the smallest volume possible. In the latter case the shear band width is set by the length scale associated with the plastic strain gradients incorporated in the constitutive model.

3. Large strain thermo-plasticity

Large strains often accompany the localized deformation. They can induce geometrical softening and this way a second source of instabilities is introduced. The condition of acoustic
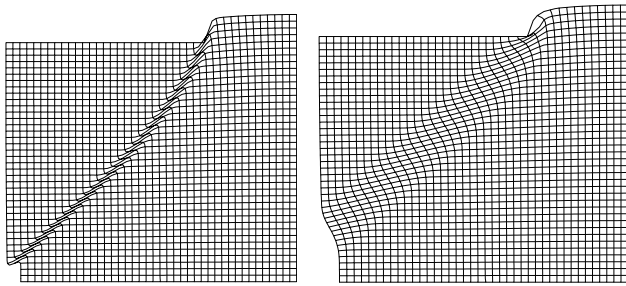
Figure 2: Plane strain simulation of vertical embankment stability using Cam-clay plasticity model: local (left) vs. gradient-enhanced (right)

tensor singularity must take into account a proper finite strain description, cf. [8]. Moreover, plastic processes involve the self-heating phenomenon and, with the increase of temperature, thermal softening is often observed in materials in addition to thermal expansion.

Therefore, focusing on metals or isotropic ductile composites, to examine complex instability phenomena without the limitation of isothermal processes, a large strain thermoplasticity model is adopted. It is based on the multiplicative decomposition of the deformation gradient into its mechanical (elastic and plastic) and thermal parts. The free energy potential is assumed in a respective additive form [9]. The classical Huber-Mises yield condition written in terms of Kirchhoff stresses is employed, together with associated flow rule, isotropic strain hardening and thermal softening. The Fourier law is introduced into the energy balance equation in temperature form, governing nonstationary heat transport.

The two-field model is implemented using the symbolic-numerical packages *AceGen* (code generator) and *AceFEM* (solution engine) for *Wolfram Mathematica* and the above-mentioned thermo-mechanical coupling effects are included. The important advantage of *AceGen* is automatic differentiation which enables an easy derivation of the tangent operator, cf. [10].

Shear banding in a plate in tension and plane strain conditions, caused by linear thermal softening, is examined. It
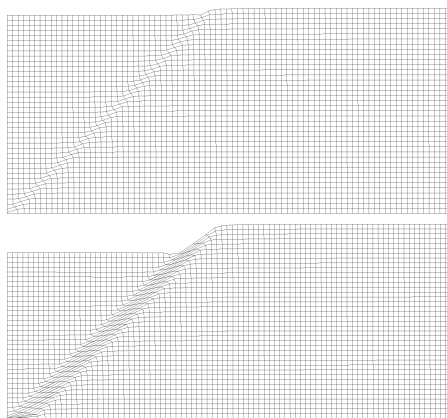

Figure 3: Influence of heat conduction on shear band in tensile plate simulated with large strain Huber-Mises thermoplasticity: adiabatic (top) vs. conductive (bottom)
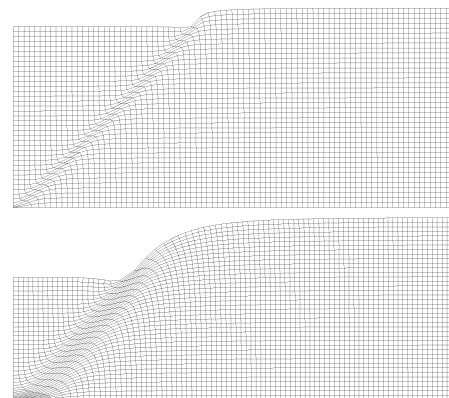

Figure 4: Influence of gradient-type averaging of relative temperature on adiabatic shear band in tensile plate simulated with large strain Huber-Mises thermo-plasticity: small internal length (top) vs. large internal length (bottom)

turns out that, for significant heat conduction, the problem of thermal softening is regularized by itself, see Fig. 3, but when one approaches the adiabatic case the results become mesh sensitive.

Therefore, in the latter case it is proposed to enhance the model with an additional diffusion-type differential equation which serves the purpose of relative temperature averaging. This gradient enhancement of the temperature field proves effective. As shown in Fig.4, the width of the shear band is determined by the internal length parameter. Moreover, it is shown that the incorporation of the two regularizing effects results in the propagation of the localization band in the specimen.

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