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Modeling of composite laminated flexible systems using absolute nodal coordinate formulation

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ABSTRACT: The current paper presents the new method of modeling the composite beam and plate structures using the absolute nodal coordinate formulation (ANCF). ANCF allows for modeling dynamics of beam and plate systems using fully spatial kinematic description and directly employing the continuum mechanics approach. Those unique features allow for modeling multi-layered structures in a straightforward manner, by imposing a simple, linear constraint equations between the layers. This allows for achieving the reliable results with limited number of degrees of freedom, due to preprocess elimination of dependent coordinates. However, it should be pointed out, that described technique should be used with care as not all of the possible configurations gives predictable and robust results.

KEYWORDS: multibody dynamics, flexible systems, composite structures, absolute nodal coordinate formulation

1. Introduction

Absolute nodal coordinate formulation (ANCF) [1] is a method for modeling flexible multibody systems undergoing large displacements and deformations. The ANCF is a nonlinear finite element method that do not include any rotational degrees of freedom in order to describe finite rotations. Instead of them the gradient vectors are used. The ANCF characteristics include the constant mass matrix, exact description of the arbitrary rigid body motions and the possibility of direct application of the constitutive equations in the material models.

The main objective of the current study is to present the ability of modeling composite laminated structures using ANCF beam and plate elements connected with linear constraint equations. These types of structures are often used in practical applications like aerospace, automotive, ship vehicles and many others [2].

2. Linear constraint equations

Using the ANCF description one can easily impose a required continuity condition at an arbitrary point on the boundary of the adjoining elements. For example, to impose C^0 continuity at the point *P* on the boundary of elements *i* and *j*, the following equation must be fulfilled:

$$\mathbf{r}_i^P - \mathbf{r}_j^P = \mathbf{0} \qquad (1)$$

where $\mathbf{r}_\alpha^P = \mathbf{S}_\alpha\left(\xi^P, \eta^P\right)\mathbf{e}_\alpha$, for $\alpha = i, j$, is the position vector of point *P* at the element α, $\mathbf{S}_\alpha$ is element α matrix of shape functions, ξ^P and η^P are the element dimensionless coordinates while $\mathbf{e}_\alpha$ is the vector of the element nodal coordinates. When the higher order continuity is required at the point *P*, one can write the constraint equations that equates an appropriate gradient vectors. All those constraint equations are linear functions of the body coordinates and therefore, they might be eliminated at the preprocessing stage.

When the connection without any gaps between two elements is required, the Eq. (1) must be ensured at each point on the boundary.

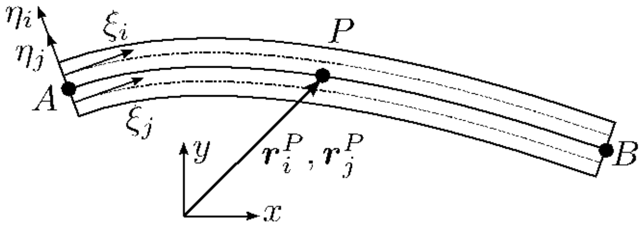

Fig. 1. Two-layer composite beam structure

If we consider a body that consists of two planar beam elements, like the model depicted in Fig. 1, one can guarantee the exact connection by imposing the following equations:

$$\begin{cases} \mathbf{r}_i^A = \mathbf{r}_j^A, & \mathbf{r}_i^B = \mathbf{r}_j^B, \\ \mathbf{r}_{i,x}^A = \mathbf{r}_{j,x}^A, & \mathbf{r}_{i,x}^B = \mathbf{r}_{j,x}^B, \\ \mathbf{r}_{i,y}^A = \mathbf{r}_{j,y}^A, & \mathbf{r}_{i,y}^B = \mathbf{r}_{j,y}^B. \end{cases} \qquad (2)$$

where *A* and *B* are points at both ends of elements boundary and $\mathbf{r}_{\alpha,\beta} = \partial \mathbf{r}_\alpha / \partial \beta$ for $\beta = x, y$ are gradient vectors of the element α.

In order to achieve exact connection between layers, in the case of the three-dimensional beam element [1], additional equations $\mathbf{r}_{i,z}^A = \mathbf{r}_{j,z}^A$ and $\mathbf{r}_{i,z}^B = \mathbf{r}_{j,z}^B$ must be included, while in the case of composite created with fully parameterized plate elements [3], the position and gradient vectors for *x*, *y* and *z* must be equated at four boundary corners. It should be pointed out, that the number of additional constraint equations is equal to the number of the single element coordinates. Thus, the multi-layer element and the single element have the same number of degrees of freedom.

Then the multi-layer elements must be assembled to create the flexible body. Figure 2 shows two possible element arrangements for the beam elements, however the similar approach might be used with the plate elements. The connection type 1, shown in Fig. 2a, consider a multi-layer elements as a separate components that are then connected by assembly conditions (which are usually placed in the middle of elements) and the standard boundary conditions are added at body interfaces. In this assembly type, the nodal points are not directly connected. It is worth noting that the connection type 1 results in the kinematic description that corresponds to the usage of the multi-layer material model [4]. However, in case of the composite laminated bodies depicted in Fig. 2, each element might use any available material model. On the contrary, the connection type 2 shown in Fig. 2b, connects with the layer connectivity conditions element layers that follow the standard assembly procedure (with shared nodes). Despite that both connection types result in the same number of degrees of freedom as the single layer body, they kinematic descriptions are not equivalent. This is due the fact that the adjacent elements in longitudinal direction are connected at different points. This results, for example, in significant differences in results of the linear modal analysis.

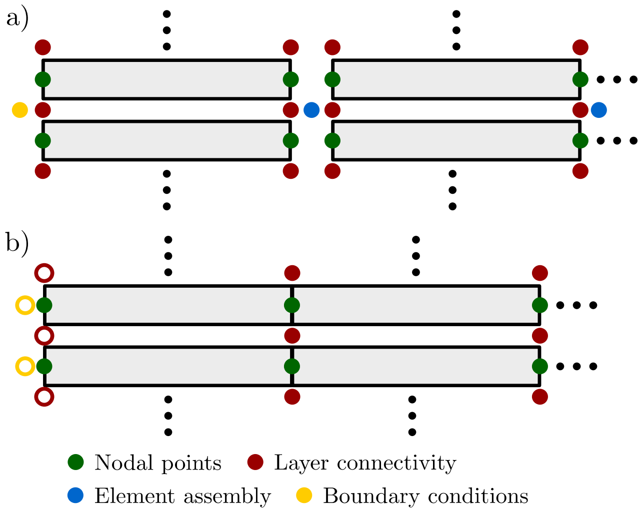

Fig. 2. Composite laminated body, a) connection type 1, b) connection type 2

In addition, in the case of the connection type 2, the layer connectivity and boundary conditions at the body boundary (points that are marked with hollow circles in Fig. 2b) must be selected with care to avoid singularities. However, the connection type 2 allows for a reduction of the continuity order between layers. The connection type 1 is much more sensitive on such modifications, but in either case this reduction must be made with the great attention. The reduction of the continuity order might be beneficial in case of using the material models that requires the reduced integration, like in the case of the incompressible material models [5], where the application of high order continuity between layers (higher then C^0) causes serious numerical issues.

3. Numerical results

Presented approach was tested with several numerical examples of modal, static and dynamic analysis and the good agreement with the reference results was observed.

In case of properly applied connection type 1, the results agree very well with single layer models, when the uniform material model is used across the layer in the multi-layer body. In addition, the comparison with the composite material model [4] provides appropriate outcomes. Moreover, the correct application of the connection type 2 with reduced layer continuity produces an acceptable solutions, as can be seen in Fig. 3. However, in this case one should pay special attention in order to create a reliable model and a more detailed analysis is advised.

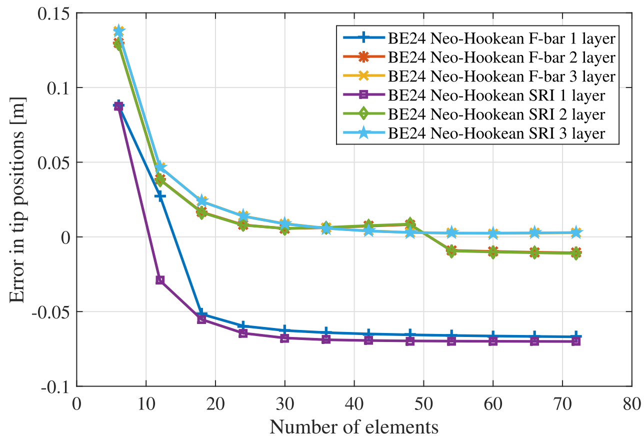

Fig. 3. Results of the static tests of the clamped-free beam under gravitational force with type 2 connection, C^0 continuity and Neo-Hookean material model. BE24 – standard spatial ANCF beam element with 24 coordinates, SRI – selective reduced integration, F-bar – strain projection with so-called F-bar method [5]. It can be noticed that for more than 2 layers the results converge to proper solution

4. Conclusions

In current paper the multi-layer flexible structures build with ANCF elements are proposed and examined. As result two types of linear layer connectivity conditions are shown and applied to simple test examples.

1) Application of linear constraints allow for reduction of the number of system degrees of freedom allowing for reasonably efficient computations.
2) Connection type 1 is universal and can be applied to variety of locking free formulations.
3) Connection type 2 can be beneficial in some special applications, but a superior attention is required when this approach is used.

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