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Dynamic buckling of thin plate subjected to compressive harmonic load

Mehdi Bohlooly Fotovat, Tomasz Kubiak
Katedra Wytrzymałości Materiałów i Konstrukcji, Politechnika Łódzka
email: mehdi.bohlooly-fotovat@p.lodz.pl, tomasz.kubiak@p.lodz.pl

ABSTRACT: The aim of the performed research was to check the possibility of employing the tools used in the analysis of the stability of motion of solids for the analysis of the dynamic stability of thin-walled square plates subjected to in-plane compressive periodic harmonic load and find the range of its chaotic behaviour. Using Hamilton's principle, differential equations of motion describing the plate deflection in time with the assumption of a two-parameter deflection function were derived. The Runge-Kutta method was used to solve them. The obtained time courses of plate deflections allowed us to determine maximal deflection, phase portraits, Poincare maps and Lapunov exponents, and on their basis to determine areas of unstable and chaotic behaviour. The obtained results were compared with the results from FEM. The determined range of chaotic behaviour was compared with the areas determined on the basis of the dynamic buckling criteria given by Volmir or Budianski-Hutchinson. The comparison of the results showed the advantages of the methods used in the dynamics of motion and the need for a different view of static and dynamic behaviour at impulse loads of finite duration and harmonics of infinite duration.

KEYWORDS: thin plate, dynamic buckling, dynamic response, stability analysis

1. Introduction

The analysis of the stability of motion and the dynamic buckling were separately very widely investigated and published in world-wise literature. Nevertheless, in world-wise literature, there are only a few papers (e.g. [1-4]) dealing with investigations of dynamic buckling of plates or plate structures subjected to pulse loading, where were used one or more methods well known for the stability of motion (e.g.: phase portraits, Poincare maps, FFT analysis and the largest Lyapunov exponents). Their authors analysed the transition from harmonic to chaotic vibration based on analysis of parametric vibration of flexible squared plates [1], checked the influence of damping on the behaviour of rectangular plates subjected to in-plane compression [2], analysed dynamic buckling of the plates under thermal and mechanical pulse [3], or determined critical pulse loading for columns with intermediate stiffeners [4].

The deep literature overview, which is only very shortly presented above allows summing up that there is still an area for dynamic buckling investigations employing well-known methods used in dynamic response and stability of motion analysis.

The Authors decided to check the suitability of methods used in stability of motions of rigid-body analysis and investigate the behaviour of thin steel square plates determining its stable, non-stable or chaotic behaviour and estimate the load leading to dynamic buckling.

2. Object of analysis and methods of solution

The thin square plate ($a = b = 100$ mm, $h = 1$ mm) made of steel ($E = 200$ GPa, $\nu = 0.3$ and $\rho = 7850$ kg/m3), which is simply supported on all edges (Fig. 1) and loaded by harmonic load $F_x(t) = F_0 + F_A\cos(\omega t)$ have been considered.

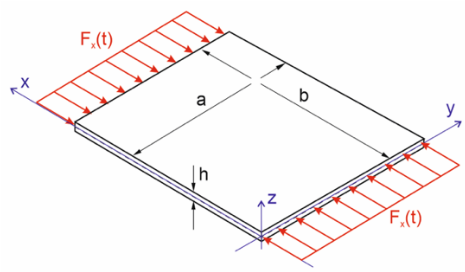

Fig. 1. Considered plate with assumed coordinate system, dimensions, and load

Analytical-numerical method and FEM were employed to solve the problem.

In the analytical-numerical method, the classical plate theory has been considered. The differential equations of equilibrium of square plate obtained from Hamilton's principle were used to derive differential equations of motion, where the following plate deflection function was assumed:

$$w = f_{11} sin\frac{\pi x}{a} sin\frac{\pi y}{a} + f_{m1} sin\frac{m\pi x}{a} sin\frac{\pi y}{a} \quad (1)$$

The two cases of deflection function were considered, where the number of half-waves m in eq. (1) was set as 2 or 3. The determined set of equations of motion describing the deflection of the plate subjected to periodic harmonic load are presented as Eqs. (2) and (3), in accordance with the assumed deflection function with m= 2 and 3, respectively.

$$\begin{cases} \frac{1}{\omega_{110}^2}\ddot{\xi}_1 + \xi_1 - \frac{p_x}{p_{cr1}}(\xi_1 + \xi_1^*) + a_1\xi_1^3 + a_2\xi_1\xi_2^2 = 0 \\ \frac{1}{\omega_{210}^2}\ddot{\xi}_2 + \xi_2 - \frac{p_x}{p_{cr2}}(\xi_2 + \xi_2^*) + a_3\xi_1^2\xi_2 + a_4\xi_2^3 = 0 \end{cases} \quad (2)$$

$$\begin{cases}\frac{1}{\omega_{110}^2}\ddot{\xi}_1+\xi_1-\frac{p_x}{p_{cr1}}(\xi_1+\xi_1^*)+b_1\xi_1^3-b_2\xi_1^2\xi_3+b_3\xi_1\xi_3^2=0\\ \frac{1}{\omega_{310}^2}\ddot{\xi}_3+\xi_3-\frac{p_x}{p_{cr3}}(\xi_3+\xi_3^*)-b_4\xi_1^3+b_5\xi_1^2\xi_3+b_6\xi_3^3=0\end{cases} \qquad (3)$$

where: ξ is nondimensional amplitude deflection normalised by plate thickness, ω_{i10} – natural plate frequencies corresponding to i = 1,2 or 3 half-waves in longitudinal direction $p_x = F_x(t)$, p_{cri} – static buckling load with i = 1,2 or 3 half-waves in longitudinal direction, $a_1 = b_1 = \frac{3}{8}\mu$, $a_2 = \frac{71271}{33800}\mu$, $a_3 = \frac{71271}{211250}\mu$, $a_4 = \frac{51}{100}\mu$, $b_2 = \frac{3}{2}b_1$, $b_3 = \frac{213}{225}\mu$, $b_4 = \frac{3}{400}\mu$, $b_5 = \frac{639}{5000}\mu$, $b_6 = 82b_4$ and $a_1 = \mu = (1-\nu^2)$.

The equation of motion (2) and (3) was solved by the Runge-Kutta method. In each case, the integration has been performed for 100 periods of excitation with zero initial conditions (for t = 0 ξ=0 and $\xi_{,t}$=0).

The FEM model have been prepared in ANSYS software. The 400 shell elements have been used. The deflection in time of initially imperfected plate was found by performing geometrical nonlinear transient analysis taking full mass and stiffness matrices. The plate behaviour was tracked in time equal to 100 excitation periods.

3. Results and remarks

In the analytical-numerical method, at the very beginning, the deflection function was considered for cases when 1 and 3 half-waves can exist in the longitudinal direction (Eq. (3)).

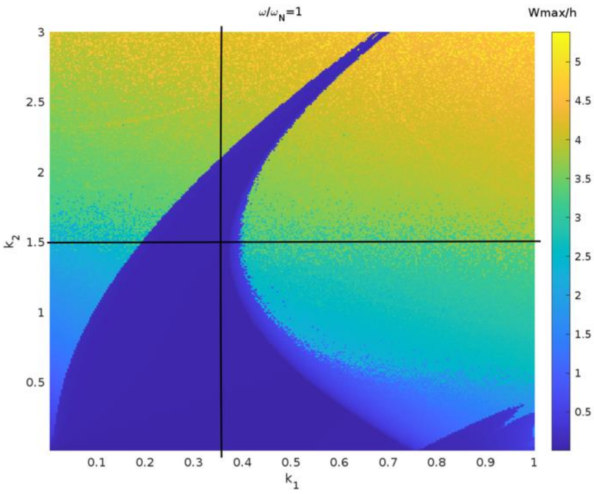

Fig. 2. Plate deflection in centre point for different load cases $k_1 = F_0/F_{cr}$ and $k_2 = F_A/F_{cr}$

It is well known, that in the case of static load one half-wave in a compressed square plate appears due to buckling, and adding the second term of the deflection equation (c.f. Eq. (3)) for m = 3 improves the solution and brings it closer to the real behaviour for loads greater than the critical loads. Nevertheless, the first solutions have shown that such an assumption is wrong. The map in Fig. 2 presents possible behaviour (stable - blue area) in comparison to the FEM solution (Fig. 3) for load defined as $k_1 = F_0/F_{cr}$ = 0.36 and $k_2 = F_A/F_{cr}$ = 1.5 gives different results. FEM results show chaotic behaviour with highest nondimensional deflection close to 2.

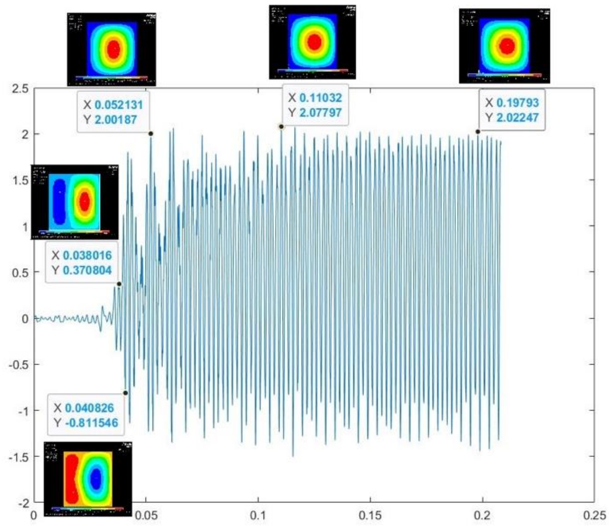

Fig. 3. ANSYS solution nondimensional deflection at centre point (axis Y) vs. time [s] (axis X)

The analysis of in-time plate behaviour has shown that two half-waves appear, which indicates that in the case of dynamic load the other behaviour could be expected. The solution becomes better if the deflection function in form as in Eq. 2 was assumed, then analytical-numerical results are closer to those from FEM (see. Fig.4).

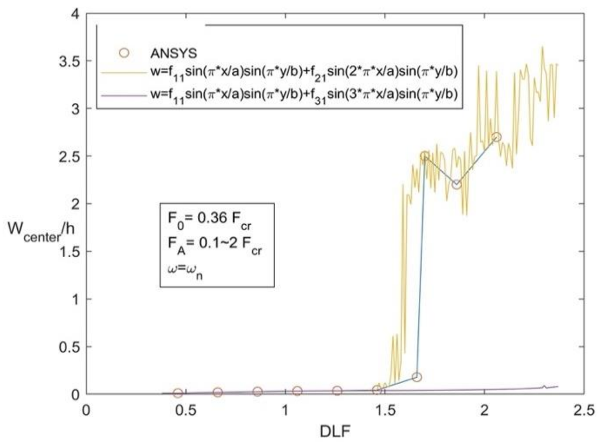

Fig. 4. Nondimensional deflection vs. DLF = $(F_0+F_A)/F_{cr}$

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