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Modeling and solving problems of mechanics with variable boundaries

Ivan Dyyak[1], Ihor Makar[1], Ihor Prokopyshyn[2], Mirosław Wesolowski[3]

[1]Department of Applied Mathematics, Ivan Franko National University of Lviv, Ukraine
[2]Pidstryhach Institute for Applied Problems in Mechanics and Mathematics, National Academy of Sciences of Ukraine,
[3]Department of Structural Mechanics, Koszalin University of Technology, Poland
email: ivan.dyyak@lnu.edu.ua., ihor.makar@lnu.edu.ua, ihor84@gmail.com, miroslaw.wesolowski@tu.koszalin.pl

ABSTRACT: We consider two problems: a contact problem within the linear theory of elasticity with an unknown contact zone and a penetration problem within the elastoplastic theory with a variable contact zone. We solve the problem of imperfect contact of an elastic body and a thin body, which is the coating of the other elastic body. Based on the domain decomposition method and FEM for massive and thin-walled objects, numerical results were obtained that confirm the effectiveness of the proposed approach. We present a novel computational framework for simulating the intricate process of screw drilling into wood. By combining cutting-edge FEM techniques, innovative modeling strategies and advanced software functionalities, this research offers a valuable tool for optimizing industrial processes.

KEY WORDS: Contact of elastic bodies, thin coatings, nonlinear Winkler layers, Tymoshenko-type shells, domain decomposition methods, screw drilling into wood, elasto-plasticity, finite element method

1. Numerical Analysis of Contact Problem

We assume that on the interface between the elastic body Ω_1 and the thin coating Ω_2 of the body Ω_3 the conditions of unilateral frictionless contact through a nonlinear Winkler layer are imposed, whereas on the interface between the body Ω_3 and its coating Ω_2 the conditions of bilateral contact through the other nonlinear Winkler layer are satisfied (Fig. 1). Suppose that the boundaries $\Gamma_\alpha = \partial\Omega_\alpha$, $\alpha = 1,2,3$, of all bodies are Lipschitz. In $\mathbf{R}^2$, we introduce a Cartesian coordinate system x_1, x_2.

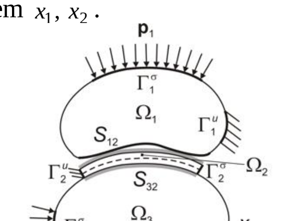

Fig. 1 Contact of the bodies in the presence of a coating and Winkler layers.

At a point $\mathbf{x} = (x_1, x_2)^\top$, the stress-strain state of each massive body Ω_α, $\alpha = 1,3$, is determined by the displacement vector $\mathbf{u}_\alpha(\mathbf{x}) = u_{\alpha i}(\mathbf{x})\mathbf{e}_i$, the strain tensor $\hat{\boldsymbol{\varepsilon}}_\alpha(\mathbf{x}) = \varepsilon_{\alpha ij}(\mathbf{x})\mathbf{e}_i \otimes \mathbf{e}_j$, and the stress tensor $\hat{\boldsymbol{\sigma}}_\alpha(\mathbf{x}) = \sigma_{\alpha ij}(\mathbf{x})\mathbf{e}_i \otimes \mathbf{e}_j$ whose components satisfy the following equations of the classical theory of elasticity under the conditions of plane deformation:

$$\sum_{j=1}^{2} \frac{\partial \sigma_{\alpha ij}(\mathbf{x})}{\partial x_j} = 0, \quad i = 1,2, \quad \mathbf{x} \in \Omega_\alpha \qquad (1)$$

$$\sigma_{\alpha ij}(\mathbf{x}) = \delta_{ij}\lambda_\alpha\left(\varepsilon_{\alpha 11}(\mathbf{x}) + \varepsilon_{\alpha 22}(\mathbf{x})\right) + 2\mu_\alpha \varepsilon_{\alpha ij}(\mathbf{x}), \quad i,j = 1,2, \quad \mathbf{x} \in \Omega_\alpha \qquad (2)$$

$$\varepsilon_{\alpha ij}(\mathbf{x}) = \frac{1}{2}\left(\frac{\partial u_{\alpha i}(\mathbf{x})}{\partial x_j} + \frac{\partial u_{\alpha j}(\mathbf{x})}{\partial x_i}\right), \; i,j = 1,2, \; \mathbf{x} \in \Omega_\alpha, \; \alpha = 1,3 \qquad (3)$$

where $\delta_{ij} = \{1,\, i = j\} \vee \{0,\, i \neq j\}$ is the Kronecker symbol and $\lambda_\alpha(\mathbf{x})$, $\mu_\alpha(\mathbf{x})$ are Lamé parameters.

To describe the stress-strain state of the coating Ω_2, we use the equations of Tymoshenko-type shell theory. For this purpose, in the thin body Ω_2, we introduce a local curvilinear coordinate system ξ_1, ξ_2, ξ_3 related to its middle surface Ω_2^*. Assume that the coating Ω_2 is a cylindrical shell infinite in the direction ξ_2.

By v_{21}, w_2, and γ_{21} we denote the tangential displacement, the normal displacement, and the angle of rotation, respectively. Further, we denote the strains by ε_{211}, ε_{213}, and χ_{211}. The forces and moment in the shell are denoted by T_{211}, T_{213}, and M_{211}, respectively. Suppose that on the boundaries of the massive bodies we impose kinematic or static boundary conditions. Assume that $S_{12} \subset \Gamma_1$ is the possible contact zone between the body Ω_1 and the coating Ω_2, and that $S_{21} \subset \Gamma_2$ is the possible contact zone between the coating Ω_2 and the body Ω_1. On the boundaries S_{12} and S_{21} we impose the conditions of unilateral contact through the nonlinear Winkler layer.

We have shown that the original contact problem is equivalent in weak sense to the problem of minimization of the nonquadratic functional

$$F(\mathbf{u}) = \frac{1}{2}A(\mathbf{u},\mathbf{u}) + J(\mathbf{u}) - L(\mathbf{u}) \to \min_{\mathbf{u}\in V_0} \qquad (4)$$

in the space V_0 and, in addition, the minimization problem (4) is equivalent to the nonlinear variational equation

$$F'(\mathbf{u},\mathbf{u}^*) = A(\mathbf{u},\mathbf{u}^*) + J'(\mathbf{u},\mathbf{u}^*) - L(\mathbf{u}^*) = 0\,,\ \forall \mathbf{u}^* \in V_0\,,\ \mathbf{u} \in V_0 \quad (5)$$

Thus, the solution of the original contact problem is reduced to the solution of the nonlinear variational equation (5) in the space V_0. We consider closed subspaces $V_\alpha^0 = \{\mathbf{u}_\alpha \in V_\alpha :\ \mathbf{u}_\alpha = 0 \text{ on } \Gamma_\alpha^u\}$, $\alpha = 1,2,3$, of these spaces and consider a reflexive Banach space $V_0 = V_1^0 \times V_2^0 \times V_3^0 = \{\mathbf{u} = (\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)^* :\ \mathbf{u}_\alpha \in V_\alpha^0,\ \alpha = 1,2,3\}$ as the direct product of the spaces V_α^0, $\alpha = 1,2,3$. To solve the nonlinear variational equation (5) of the contact problem, we use the following implicit nonstationary iterative method [1]:

$$G^k(\mathbf{u}^{k+1},\mathbf{u}^*) = G^k(\mathbf{u}^k,\mathbf{u}^*) - \rho^k\left[A(\mathbf{u}^k,\mathbf{u}^*) + J'(\mathbf{u}^k,\mathbf{u}^*) - L(\mathbf{u}^*)\right]$$

$$\forall \mathbf{u}^* \in V_0\,,\ k = 0,1,\ldots .$$

We had also performed software implementation of the domain decomposition algorithm using the FEM to solve the elasticity problems in massive bodies and the one-dimensional FEM with bubble basis functions of high order to solve the problem of the Tymoshenko-type shell theory.

The proposed method was used for the numerical investigation of the problem of contact interaction between two elastic bodies with surface groove in the presence of a thin elastic coating and the nonlinear Winkler layers. We had performed the comparison of the numerical solutions obtained by the domain decomposition algorithm based on the use of the Tymoshenko-type shell theory to model the stress-strain state of the thin coating with the solutions obtained by the DDM in which the classical elasticity theory was used to model the coating.

The solutions of the problems with an unknown contact zone and penetration problems are more important nowadays than ever.

2. Finite element simulation of screw drilling into wood

The calculation and analysis of torque and the local stress-strain state in wood material is crucial for optimizing screw design and improving industrial processes [2, 3].

We propose a unique approach to simulate the screw drilling process by sequentially generating 'screw drilling frames'. Each of such frames represents the wood material at certain screw penetration level. As screw drilling progresses, a larger contact area between the wood and the screw is developed. The seamless transition between frames involves setting prescribed horizontal displacements and transferring stresses and residual plastic strain to the subsequent frame as initial conditions in FEM simulation.

One half of the model is considered for each screw drilling frame due to the symmetry conditions. The finite element mesh and the boundary conditions of a screw drilling frame are illustrated in the figure 1.

A dynamic meshing using 2D quadratic triangle elements is adopted to discretize the wood medium precisely, constantly adapting the mesh and boundary conditions at each penetration level to capture the nuances of the interaction.

Referring the figure 2, the boundary conditions are defined as following:

- the symmetry boundary conditions $d_x = 0$ on edge AE;
- the prescribed horizontal displacements d_x on boundary edges from A to B (i.e., the contact between screw surface and wood);
- fixed boundary conditions $d_x = 0, d_y = 0$ on edges BC, CD and DE.

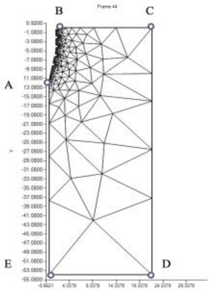

Fig. 2. Finite element mesh and the geometry of the screw drilling frame

In present study the wood material is modelled as elasto-plastic orthotropic according to the Von Mises yield criterion. Using the principle of virtual work, the stress-strain state in the screw frame $\Omega \subset R^2$ can be described as [4]:

$$\int_\Omega \delta\boldsymbol{\varepsilon}^T\boldsymbol{\sigma}\, d\Omega\ -\ \int_\Omega \delta\boldsymbol{d}^T\boldsymbol{f_m}\, d\Omega - \int_\Gamma \delta\boldsymbol{d}^T\boldsymbol{f_h}\, d\Gamma\ = 0 \quad (6)$$

where Γ is a boundary of Ω, $\boldsymbol{f_m}$ – mass forces, $\boldsymbol{f_h}$ – surface forces, $\boldsymbol{d}, \boldsymbol{\varepsilon}, \boldsymbol{\sigma}$ – displacements, strains and stresses, respectively.

The nonlinear equilibrium equation (6) is further solved for each screw frame (see figure 2) by means of Newton-Raphson iterations [4], resulting in torque at corresponding screw penetration level. The torque is calculated using the stresses at nodes on frame edge AB (figure 2) as output of FEM simulation.

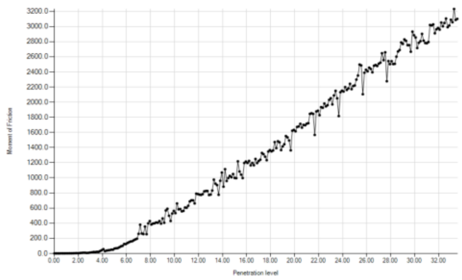

Fig. 3. The calculated torque per screw penetration level

The developed software has been successfully tested by engineers to calculate the resultant torque that was well agreed with experiments and valuable in making decisions on the screw geometry optimization. The present study yielded qualitative results comparable to those observed in real experiments.

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