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The damped thin plate behaviour undr in-plane compressive excitation

Przemysław Perlikowski[1], Mehdi Bohlooly Fotovat[2], Tomasz Kubiak[2]
[1]Devision of Dynamics, Lodz University of Technology
[2]Department of Strenght of Materials, Lodz University of Technology
email: przemyslaw.perlikowski@p.lodz.pl, mehdi.bohlooly-fotovat@p.lodz.pl, Tomasz.kubiak@p.lodz.pl

ABSTRACT: The dynamics of a rectangular plate with initial deflection excited by an external time depended periodically varied force. The plate under analysis is made of isotropic material. It was assumed that the plate is simply supported on all edges and has initial geometrical imperfection perpendicular to its surface. Two models describing the plate behavior were created. The first model (reduced) is based on derived partial differential equations - the one-degree-of-freedom Mathieu equation. The second (full) model was prepared in ANSYS software. The equivalence of the full model with the reduced one was conformed initially by the researchers. A good agreement is also observed for values of stress obtained from full and reduced models. The results from reduced model were obtained hundreds time faster than from the full one. It was also found that the Mathieu equation was suitable for investigating the dynamics of the rectangular plate across various parameters and for studying the sensitivity to initial conditions. The origins, mathematical solutions, and stability analysis of the Mathieu equation were explored by the authors, highlighting its role as a fundamental tool for understanding the response of the original system.

KEY WORDS: Dynamic responce, thin rectangular plate, damped system

1. Introduction

The response of thin structures to dynamic load was and still is the object of investigation by many scientists [1, 2]. The paper focuses on the thin plate subjected to harmonic compressive load in its plane. As it is well known, the compressive in-plane loads lead to the buckling phenomenon, which reduces the plate's stiffness in the post-buckling range [3]. Considering the above account, in the case of harmonic load, the change in the range from pre-buckling to post-buckling and vice versa in time could occur. So, it means that the stiffness of the considered plate depends on the mean value and amplitude of the assumed load cycle and could change over time.

The authors of this paper decided to perform research to analyze the possibility of employing well-known methods in the stability of motion in the case of stability of structure by analyzing how to change the deflection of the plate under in-plane harmonic load. The papers [4, 5] present the results of their research dealing with the plate for which the damping was not considered. Assuming non-zero damping in the considered structure gives the possibility of analysis in a mathematical way of different scenarios of plate behavior in case of load corresponding to the load leading the nondamped plate to the non-periodic behavior. This approach could make it possible to use the phenomenon of buckling in thin plates loaded harmonically in their plane to design various control systems, MEMS systems, or damage detection systems (SHM - Structural Health Monitoring). Mathieu's equations have been employed to analyze the harmonically loaded plate response.

In many applications, the Mathieu equation can be solved analytically [6]; however, it is desired to detect all existing solutions in the assumed range of initial conditions and to show complex responses for large values of forcing terms.

To validate results obtained from Mathieu equation the FEM model of the plate was created. The eigen values analyses determining buckling load and natural frequencies and transient dynamic analyses allowing to check in-time plate behavior (its deflection and stress distribution) have been performed. The results obtained from one-degree of freedom Mahieu equation have been compared with results from FEM simulation (c.a. 2400 DoF). The acceptable agreement has been obtained, what prove that the reduced model is good enough to perform deep parametric study.

2. Model of the system

The plate under consideration is a rectangular steel plate of size 100 [mm] x 100 [mm] and thickness h = 1 [mm] subjected to harmonic compressive load. The Young modulus, Poisson ratio and density have following values: 2e5 [MPa], 0.3 [−] and 7850 [kg/m^3]. The harmonic load was defined in the following way:

$$F_x(t) = \gamma_1 F_{cr} + \gamma_2 F_{cr} \cos \omega t \tag{1}$$

where: $\gamma_1 = F_0/F_{cr}$ and $\gamma_2 = F_A/F_{cr}$, are non-dimensional mean value and amplitude of the harmonic load, respectively. F_0 is a mean value, F_A is an amplitude, and F_{cr} is a static buckling load (eigenvalue).

To obtain the Mathieu equation describing the in time plate deflection the classical plate theory with von Karman geometrical nonlienarities were consider. The plate deflection was assumed in form:

$$w(x,y) = f \sin\frac{\pi x}{a} \sin\frac{\pi y}{a} \tag{2}$$

Employing Galerkin procedure by deriving residuals, substituting deflection function of Eq. (2), imperfection and determined stress function into equation of motion supplemented compared to those from work [4] the Mathieu equation described the system was obtain in the following form:

$$\ddot{\chi} = -\zeta c_c \dot{\chi} - a_1 \chi + a_2(\gamma_1 + \gamma_2 \cos \Theta t)(\chi + \xi) - a_3 \chi^3 \quad (3)$$

where $c_c = 2\sqrt{a_1 + a_2\gamma_1}$ is critical damping and $\chi = f/h$; and the values of parameters are as follow: a_1 = 9090697, a_2 = 125727 F_{cr}, a_3 = 3102200. In this study parameters are varied in the following ranges: $\gamma_1 \in (0.0, 1.0)$, $\gamma_2 \in (0.0, 2.0)$, while other parameters are fixed and have following values: Θ = 3015, F_{cr} = 72.3, ξ = 0.01. The damping in the system is small to be close to real thin square plate, hence it is assumed that ζ is equal to 1% of critical damping: ζ = 0.01.

Taking the amplitude of plate deflection ($f = \chi h$) from the solution of Mathieu Equation in given moment of time and having the stress function the stress state at each point (x, y) of plate can be easilly calculated.

3. Results

The results of calculation obtained from reduced and full model in form of phase portraits and stress distributions maps are presented in Figs. 2 – 3. As it is well visible very good agreement have been obtained if the phase portrais are compared. The stress distribution comparison is in a satisfactory level considering the differences in DOF number in both models.

a) b)

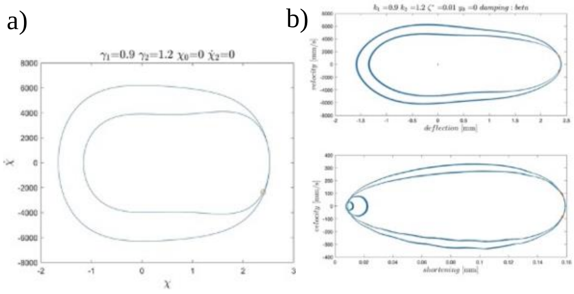

Fig. 1. Phase portraits obtained from reduced (a) and full (b) model

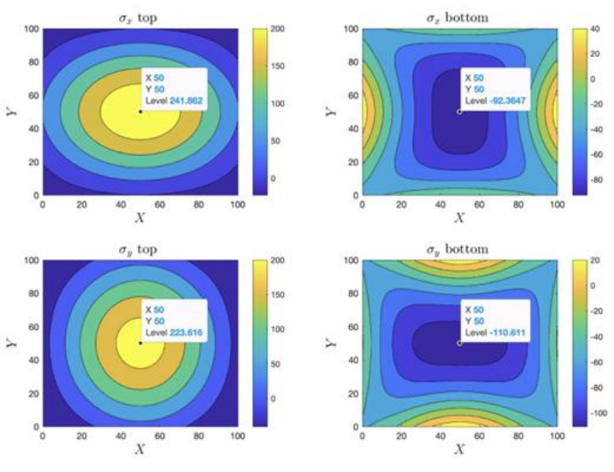

Fig. 2. Stress distribution from reduced model for γ_1 = 0.1 and γ_2 = 1.25, and plate deflection f = 1.44 mm

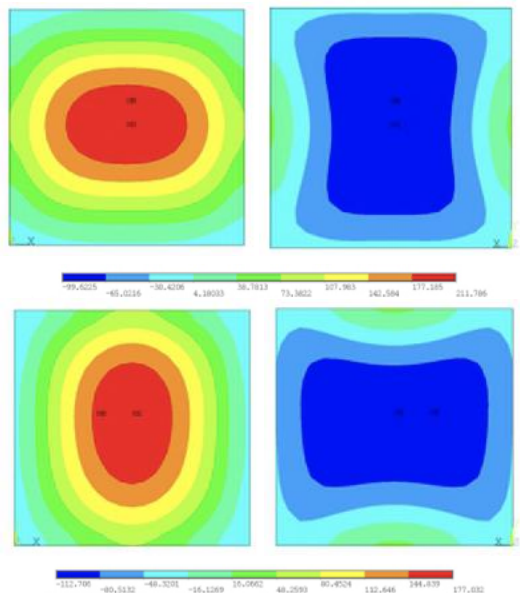

Fig. 3. Stress distribution from FEM for γ_1 = 0.1 and γ_2 = 1.25, and plate deflection f = 1.44 mm

4. Conclusions

The plate's PDE equation is reduced to the one-degree-of-freedom Mathieu equation. To analyze the system response the sampled-based method and path-following have been utilized. Both tools allow us to investigate the dynamics of the system and show solutions, bifurcations and structure of the phase space. Then, the solutions obtained based on the Mathieu equation are compared with the full FEM model. It shows that both methods give qualitatively the same results and the differences appear for high DLF (i.e., $\gamma_1 + \gamma_2$) values.

Summarizing, the results prove that the reduction of the square plate model to the Mathieu equation gives reliable information about the system's dynamics. It significantly speeds up simulations and allows for investigation of the wide range of the system's parameters and initial conditions contrary to the FEM analysis where calculations are very time-consuming. The sampled-based method gives the information where the system is multistable and based on basins of attraction it is possible to predict if the solution is robust.

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