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Analytical and numerical approach for the 2-DOF helical gear model with time-varying mesh stiffness

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ABSTRACT: The purpose of this study was to analyze the parametric vibration of a helical gear system with variable mesh stiffness using the multiple time scale method. Two linear models were assumed for the study: a two-degree-of-freedom model for numerical analyses and a reduced one-degree-of-freedom model for determining the analytical solution. The 2-DOF model reduction by eliminating angular motion simplified the problem to the 1-DOF model, enabling the determination of an analytical solution at a lower computational cost due to reduced computation time. Time complexity is particularly important for performance computing. The results of simulations performed using both analytical and numerical solutions were sufficiently coincident to be comparable in quality. The overall conclusion suggests that there are significant advantages to using an analytical solution due to its precision and relatively low computational cost. Although obtaining an analytical solution is more time-consuming, it reduces the possibility of errors that can occur with numerical methods. Additionally, the analytical solution needs to be computationally determined only once, unlike the numerical solution, which must be recalculated for each set of system parameters.

KEYWORDS: vibrations, gear, nonlinear, mesh, stiffness, dumping

1. Introduction

The current research on gear dynamics and vibration analysis has advanced significantly, focusing on time-varying mesh stiffness and nonlinear relationships between mesh stiffness and dynamic forces, especially in spur gears as explored in [1]. Studies have also examined the impact of additional degrees of freedom, revealing diverse behaviors in non-linear vibrations [2]. Shaft stiffness's role in gear vibration characteristics has been highlighted, showing its effect on resonance curves and specific conditions that prevent chaotic vibrations [3, 4]. Comparisons between elliptical and circular gears demonstrate lower vibration amplitudes in elliptical gears, linked to interactions between eccentricity, stiffness, and load [5]. This study aims to investigate parametric vibrations using both numerical and analytical methods, employing two helical gear models: the DDOF (2-DOF) model for numerical simulations and the SDOF (1-DOF) reduced model for analytical solutions. The goal is to understand amplitude values related to parametric vibrations due to variable gear mesh stiffness.

2. Proposition of the model

Mentioned physical models of the study are provided by relevant sets of equations of motion, schematics and symbols descriptions. In the Fig. 1, model of the 2-DOF helical gear system is visible. Dynamics of the system is described by equations of motion in equation (1) and (2).

$$J_1\ddot{\varphi}_1 + r_{b1}c(t)\dot{x} + r_{b1}k(t)x = T_1 \tag{1}$$

$$J_2\ddot{\varphi}_2 - r_{b2}c(t)\dot{x} - r_{b2}k(t)x = -T_2 \tag{2}$$

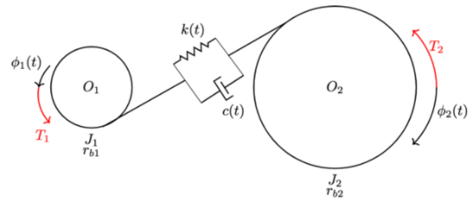

Figure 1. Model of the 2-DOF helical gear system

x - relative displacement,
$\dot{x}$- velocity,
r_{b1} - pinion radius,
r_{b2} - gear radius,
$c(t)$ - variable damping.
$k(t)$ - time varying mesh stiffness,
J_1 - mass moment of inertia of the pinion,
J_2 - mass moment of inertia of the gear,
φ_1 - first degree of freedom associated with the rotational motion of the gear wheels;
φ_2 - second degree of freedom associated with the rotational motion of the gear wheels.

The introduced model is a second ordered system of linear differential equations with varying in time coefficients. It enables to simultaneous analysis of relative vibration of gears and its rotational movement.

3. Numerical analysis

The 2-DOF model mentioned in the previous section is reduced to the 1-DOF model. The degree of freedom associated with the rotation of gears is not relevant to the investigation of vibrations. The correctness of the approximated analytical solution was verified before conducting the sensitivity analysis. The obtained solution was evaluated over a time range from 0 to 30 seconds and compared with numerical solutions calculated using the RK45 algorithm. The comparison results are shown in Fig. 2

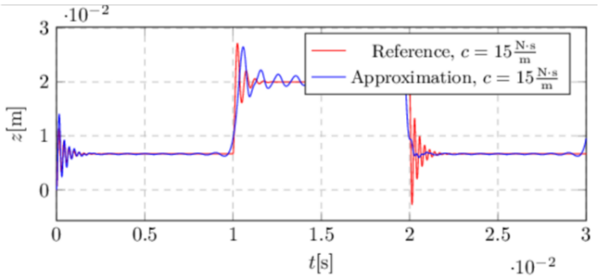

Figure 2. A comparison of reference and analytical (approximated) solution

It can be stated that the time histories of the analytical and numerical solutions are convergent. This convergence justifies the use of approximated analytical solutions for further investigations.

The numerical investigation of these approximated analytical solutions began with a stiffness analysis. The results are presented in the following plot.

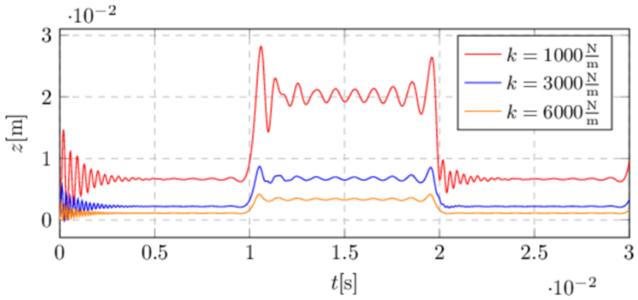

Figure 3. Time-varying displacement for different mesh stiffness values

Analysing the time-varying displacement for different mesh stiffness values leads to the conclusion that lower stiffness results in greater vibration amplitude. As mesh stiffness increases, the vibration amplitude decreases significantly. For example, with a mesh stiffness six times higher, the maximum vibration amplitude is approximately six times lower. Additionally, there is a clear periodic change in the displacement value when the second pair of teeth comes into contact, and this occurs when transitioning from a one-pair to a two-pair mesh. The time course of the displacement (tooth deflections) changes with a constant period, which correlates with the mesh stiffness, as shown in Figure 3. This pattern is due to the time when the second pair of teeth are in contact – specifically, from entering the line-of-action to exiting it. A lower displacement occurs when two pairs of teeth are in contact (within the line-of-action), resulting in higher mesh stiffness. Conversely, a higher displacement occurs when only one pair of teeth is in contact (within the line-of-action), leading to lower mesh stiffness.

The following Figure 4 shows the time-varying displacement for three different mesh damping coefficient (at a specific mesh stiffness).

It clearly shows a larger logarithmic decrement for higher damping coefficients. Subsequent amplitudes are much smaller, indicating that the system is damped faster.

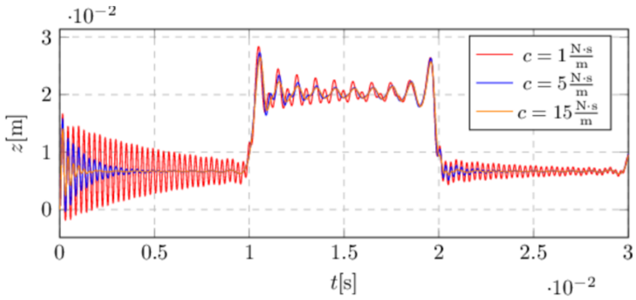

Figure 4. Time-varying displacement for different mesh damping coefficient values

The effect of varying mesh damping is particularly evident during a two-pair mesh period when both pairs of teeth are in the line-of-action. In this case, displacement values are lower, the vibration period decreases, and the number of oscillations increases. In contrast, higher displacement values occur when only one pair of teeth is in contact (within the line-of-action). In this case, the mesh damping is less effective due to the periodic excitations caused by the second pair of teeth entering and exiting the line-of-action.

4. Summary

The following conclusions can be drawn based on the conducted simulations:

1) Applying a two-degree-of-freedom (2-DOF) model reduction eliminates angular motion from the analysis, thus reducing it to a one-degree-of-freedom (1-DOF) system.
2) Simulations based on both approximate analytical and numerical solutions yield sufficiently convergent results, allowing them to be used interchangeably.
3) The analytical method allows for a qualitative assessment of the results and the use of mathematical analysis tools for verification, which is an important asset of this approach.

The simulations demonstrated the advantage of an analytical solution over numerical analysis in investigating the effect of mesh stiffness on helical gear dynamics. The procedure for obtaining the analytical solution is time-consuming but requires to be done only once. Afterward, the system response for any parameters set can be obtained quickly and automatically using appropriate software. In summary, although a numerical approach is commonly used for this type of problem, pursuing an analytical solution is recommended if sufficient research and time are available.

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